Wat. Res. Vol. 25, No. 3, pp. 341–350, 1991
Printed in Great Britain.

0043-1354/91 $3.00 + 0.00

A SIMPLIFIED PHOSPHORUS TROPHIC STATE MODEL FOR WARM-WATER TROPICAL LAKES

HENRY J. SALAS and PALOMA MARTINO
Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS), Casilla 4337, Lima 100, Perú

(First received December 1989; accepted in revised form September 1990)

Abstract—The Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS), a specialized Center of the Pan American Health Organization/World Health Organization (PAHO/WHO), has conducted a Regional Program since 1981 for the development of simplified methodologies for the evaluation of eutrophication in warm-water tropical lakes/reservoirs of Latin America and the Caribbean. The majority of the regional lakes/reservoirs appear to be phosphorus limited. A warm-water tropical lake trophic state classification system and a simplified total phosphorus model have been developed with regional data, and successfully verified against the data of other lakes/reservoirs of the African continent. The use of the phosphorus model as a predictive tool in the management and planning of water resources is considered.

Key words—tropical lakes, eutrophication, phosphorus, trophic state, lake/reservoir modeling

NOMENCLATURE

A_s = surface area (km^2)
Chl a = chlorophyll a (mg m^{-3})
CT_i = trophic state categories
K_s = overall loss rate of total phosphorus (yr^{-1})
L(N) = nitrogen load (g m^{-2} yr^{-1})
L(P) = phosphorus load (g m^{-2} yr^{-1})
μ = average
N = number of data
N_λ = total nitrogen (mg l^{-1} or g m^{-3})
P_λ = total phosphorus (mg l^{-1} or g m^{-3})
$P_{\lambda in}$ = average influent total phosphorus (mg l^{-1})
$P(X/CT_i)$ = normal distribution of probability
r^2 = determination coefficient
σ_s = standard deviation (SD)
T_ω = detention time (yr)
SD = standard deviation
SE = standard error
X = parameter (i.e. log-total phosphorus)
Y = log-normal probability distribution
$\bar{Z}$ = average depth (m).

INTRODUCTION

The explosive demographic growth in Latin America and the Caribbean over the past years, with the consequent increased demand for water resources, has accelerated the construction of artificial multipurpose reservoirs for potable and industrial water supplies, irrigation water and hydro-electric power. Many of these reservoirs and also the natural lakes in the region have suffered the consequences of the eutrophication process which has interfered with the designated uses of these water bodies and, as such, the very purposes for which the reservoirs were created.

The Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS, Spanish acronyme) located in Lima, Perú, of the Pan American Health Organization/World Health Organization (PAHO/WHO) convened a regional meeting in December 1981 in São Paulo, Brazil, to analyze methodologies for the evaluation of eutrophication and its inclusion in the planning process.

The then available simplified models (OECD, 1982), developed with data from predominantly temperate lakes, were reviewed and it was concluded that due to the fundamental differences between temperate and warm-water tropical lakes (Castagnino, 1982) these temperate lake models were not applicable to the majority of water bodies of the region. The Regional Program for the Development of Simplified Methodologies for the Evaluation of Eutrophication in Warm-Water Tropical Lakes was then initiated. A minimum data collection program to obtain the necessary data was agreed upon and the overall goals of the program were established as follows:

(i) The establishment of a trophic state classification system for warm-water tropical lakes.
(ii) The development of simplified methodologies for the evaluation of eutrophication in warm-water tropical lakes.
(iii) The development of a reliable simplified mathematical model to be applied in the management of lakes and the planning of future reservoirs.

Subsequent regional meetings were held in Brasília, Brazil (Salas, 1983), Guadalajara, México (Salas and Limón, 1985) and Sãn Juan of Puerto Rico (Salas and Martino, 1988) in which additional countries and data were incorporated into the regional program.

Table 1. Key parameters

Lake	Symbol	Average depth $\bar{Z}$ (m)	Surface area A_s (km^2)	Detention time (yr)	Phosphorus load [L(P)] (g m^{-2} yr^{-1})	Total phosphorus (P_λ) (mg l^{-1})	K_s for P_λ (yr^{-1})
Argentina							
R. Salto Grande, 1982	S_2	7.8	757	0.097	12.6	0.081	9.63
R. Salto Grande, 1981–1983	S_{13}	8.15	757	0.032	15.70	0.047	9.79
Brazil							
Brasília							
R. Descoberto, 1980	D	6.9	14.9	0.280	0.65	0.016	2.30
R. Paranoá, 1980	P	14.3	39.5	0.731	2.93	0.040	3.80
Rio de Janeiro							
R. Funil, 1978–1979	F_1	22.8	38.4	0.151	9.19	0.041	3.30
R. Funil, 1987	F_2	22.8	38.4	0.131	18.60	0.048	9.40
R. Funil, 1988–1989	F_3	21	33.3	0.081	29.3	0.066	9.0
R. Santana, 1987	S_a	2.5	4.75	0.002*	151.6	0.10	110
R. Vigario, 1987	V_i	9.8	3.85	0.008	142.9	0.084	48
R. Lajes, 1988–1989	L_a	13.6	26.0	0.760	0.795	0.018	2
Sao Paulo							
R. Americana, 1982	A_2	9.2	11.5	0.047	37.70	0.081	29.30
R. Americana, 1986	A_6	7.8	12.7	0.084	31.62	0.098	29.46
R. Atibainha, 1986	A_t	12.5	20.8	0.388	1.29	0.023	1.91
R. Barra Bonita, 1978–1979–1980	B_a	8.3	239.3	0.269	3.43	0.058	3.41
R. Barra Bonita, 1983	B_b	9.2	260	0.073	23.00	0.115	8.04
R. Barra Bonita, 1982–1984	B_c	9.3	268	0.211	7.45	0.094	3.74
R. Barra Bonita, 1986	B_d	8.6	250	0.221	6.96	0.059	9.19
R. Cachoeira, 1986	C_c	10.7	7.1	0.130	4.14	0.032	4.40
R. Guarapiranga, 1982–1983	G_2	4.9	27.8	0.238	1.40	0.052	1.30
R. Guarapiranga, 1986	G_6	4.9	23.6	0.331	1.70	0.044	4.86
R. Itupararanga, 1986	I	7.8	20.9	0.660	2.11	0.029	7.81
R. Jaguari, 1986	J	16.8	35.0	1.225	3.28	0.036	4.60
R. Paiva Castro, 1982–1983	C_2	5.7	6.7	0.059	7.30	0.040	15.10
R. Paiva Castro, 1986	C_6	5.4	4.1	0.028	5.48	0.023	8.41
R. Paraibuna, 1986	P_a	26.4	168	1.923	0.79	0.016	1.35
R. Ponte Nova, 1982–1983	N_2	8.9	21.1	0.615	0.55	0.027	0.67
R. Ponte Nova, 1986	N_6	8.3	20.5	0.806	0.47	0.025	1.02
R. Taiaçupeba, 1986	T_a	2.2	8.8	0.135	0.64	0.031	1.98
Colombia							
Laguna de Sonso	S_o	1.0	10.0	0.041	6.58	0.210	6.90
Ecuador							
R. Poza Honda, 1981	H_1	20.00	2.6	4.845	5.06	0.40	0.43
R. Poza Honda, 1982	H_2	20.14	3.8	1.574	15.10	0.20	3.11
Mexico							
Laguna de Cajititlán, 1981–1982	Ca_1	0.67	10.6	†	0.937	0.470	1.89
Laguna de Cajititlán, 1986–1987	Ca_2	1.69	14.3	†	3.011	0.400	3.53
L. Chapala, 1983–1984	Ch_1	4.20	1061	11.05	0.932	0.426	0.43
L. Chapala, 1986–1987	Ch_2	4.43	1078.5	15.94	1.508	0.680	0.44
L. Zirahuén, 1986–1987	Z	20.64	11.23	†	2.492	0.250	4.80
L. Tequesquitengo, 1986	T_e	16.00	8.0	98.5	0.046	0.023	0.067
L. Requena, 1986–1987	R	5.00	4.82	0.26	13.229	0.383	7.30
Puerto Rico							
L. La Plata, 1981–1982	L_p	10	3.07	0.09	34.90	0.220	4.57
L. Loiza, 1973–1974	L	6.14	2.425	0.054	52.91	0.330	7.34
Laguna Tortuguero, 1974–1975	T	1.2	2.24	0.133	0.30	0.010	17.50
United States of America							
L. Livingston, Tex., U.S.A., 1975	L_i	6.30	334.2	0.243	14.6	0.20	7.47
Venezuela							
L. Valencia, Jan.–Feb., 1978	V	18.3	356.0	†	2.688	0.105	1.35
Laguna Grande, 1980	L_g	3.5	1.11	0.12	13.70	0.29	5.16

ND = No data.
*Low detection time, not used in total phosphorus model.
†Without outflow, therefore has not been used in the model.
‡Low reliability.
Other participating countries (however, incomplete data sets): Cuba, Dominican Republic, El Salvador, Nicaragua, Paraguay, Perú.
R, reservoir; L, lake; HE, hypertrophic; E, eutrophic; M/E, meso/eutrophic; M = mesotrophic; O/M, oligo/mesotrophic; O, oligotrophic.

In 1990 fifteen countries/states of the region were participating, providing data from about forty lakes/reservoirs, as shown in Table 1.

The program is ongoing and the results as of March 1990 are presented herewith.

DATA ANALYSIS

The working definition utilized in the program (Salas, 1983) of a warm-water tropical lake is based on a minimum temperature of 10°C under normal conditions with a minimum annual average of 15°C. Under this definition the geographic coverage of the program extended from Lake Livingston in Texas, U.S.A., to Salto Grande in Argentina (Table 1). It is noted that high altitude tropical lakes, 3000 m above mean sea level, did not adhere to this definition and were therefore excluded from the program.

The minimum field data collection program called for monthly water quality sampling for a period of 1 for the simplified models

Nitrogen load [L(N)] ($g\,m^{-2}\,yr^{-1}$)	Total nitrogen (N_t) ($mg\,l^{-1}$)	Chlorophyll *a* surface ($mg\,m^{-3}$)	Hipolimnetic oxygen depletion rate ($mg\,l^{-1}\,month^{-1}$)	Classification	References
ND	—	—	—	M/E	Berón and Lee (1984)
ND	—	—	—	M/E	Berón and Lee (1984)
ND	0.460	ND	—	O	CAESB (1983)
—	1.260	—	—	E	CAESB (1982) and Cordeiro and Dutra Filho (1981)
98.76	0.580	8.1	—	M	FEEMA (1982)
—	—	6.0	—	E	FEEMA (1987)
212.58	0.60	10	0.5	E	FEEMA (1989)
1094	0.938	3.6	—	E	FEEMA (1989)
1019	0.836	2.7	—	E	FEEMA (1989)
9.14	0.475	2.0	0.8	O	FEEMA (1989)
ND	1.350	15.8	—	E	CETESB (1985)
356.60	2.456	32.1	5.4	E	CETESB (1987)
29.04	0.901	3.8	7.3	O	CETESB (1987)
ND	ND	10.7	—	M/E	CETESB (1985)
ND	ND	4.8	—	M/E	CETESB (1985)
ND	ND	6.2	—	M/E	CETESB (1985)
98.50	1.376	9.7	2.3	M/E	CETESB (1987)
72.1	0.790	3.6	4.4	O	CETESB (1987)
ND	0.630	20.2	—	E	CETESB (1985)
15.80	0.880	2.9	3.0	E	CETESB (1987)
32.40	0.889	8.6	3.9	M	CETESB (1987)
22.90	0.792	3.3	—	O/M	CETESB (1987)
ND	0.610	3.5	—	M	CETESB (1985)
184.80	0.831	8.0	3.4	M	CETESB (1987)
10.20	0.573	1.5	—	O	CETESB (1987)
ND	0.640	4.0	—	O/M	CETESB (1985)
7.76	0.579	6.0	3.5	O/M	CETESB (1987)
12.60	0.506	8.0	—	M	CETESB (1987)
76.98	2.470	—	0.2	HE	CVC (1988)
ND	ND	ND	—	HE	Vásconez (1983)
ND	ND	ND	—	HE	Vásconez (1983)
14.811	4.240	ND	—	HE	CEL (1985)
18.496	2.400	22.38	—	HE	CEL (1987)
3.650	0.876	5.02	—	M	CEL (1985)
5.182	1.130	8.9	—	E	CEL (1987)
17.532	0.730	1.67	0.5	E	CEL (1988)
1.335	0.645	26.4	—	M	IMTA/SARH (1986)
97.917	1.740	35.16	—	E	IMTA/SARH (1988)
197.3	0.990	16.8	—	E	EQB (1984)
232.6	1.700	ND	—	E	Quiñones (1980)
14.8	1.700	ND	—	O	USGS (1978)
53.800	1.600	23	—	E	Hydroscience (1976)
—	2.1	27.3	—	HE	DIA (1979) and Lin (1981)
256.31‡	6.56‡	—	—	E	

year in the lakes/reservoirs and their major tributaries, and it was recommended that the Clark method (Sonzogni *et al.*, 1978) be used with the tributary data to estimate nutrient loadings. Direct in-lake discharges, where significant, were estimated using export coefficients presented in Table 2, originally taken from Rast and Lee (1978) and confirmed with Brazilian data. Recommended estimates of living organism contributions (human and animal) were also provided. The above recommended criteria were not followed to the letter in all cases and, although deviations were accepted, some data sets

Table 2. Export coefficients*

Land use	Total phosphorus ($g\,m^{-2}\,yr^{-1}$)	Total nitrogen ($g\,m^{-2}\,yr^{-1}$)
Urban	0.1	0.5
Agricultural-rural	0.05	0.5
Forest	0.01	0.3

*Castagnino (1982).

Table 3. Data used in model development, range and averages

Parameter	Symbol	Units	Minimum	Geometric Mean	Maximum
Average depth	$\bar{Z}$	m	1.00	8.061	26.4
Detection time	T_ω	yr	0.008	0.283	98.5
Phosphorus load	$L(P)$	$g\,m^{-2}\,yr^{-1}$	0.046	4.461	142.9
Total phosphorus	P_λ	$mg\,m^{-3}$	10.0	66.672	680.0

had to be rejected upon review. Spatial coverage was site specific, which in some cases was limited to one in-lake station. Laboratory procedures were generally taken from *Standard Methods* (APHA, 13th, 14th, 15th, 16th editions).

For model development, 39 complete sets of data of 27 lakes/reservoirs were used, as presented in Table 1, which included diverse limnological conditions ranging from oligotrophic to hypereutrophic (trophic classification criteria discussed subsequently) and deep to shallow depths. Most of the lakes were phosphorus limited. The range for key parameters is presented in Table 3. Histograms of the distribution of depth, detention time, total phosphorus and total phosphorus load are shown in Fig. 1.

Since data were collected and analyzed by different investigators of diverse laboratories of the region over several years, a variability of the quality of data is assumed. Notwithstanding, given the wide range covered by the data and the random nature of quality variability, it is expected that this would not significantly bias the general relationships obtained.

Data were transformed to their natural logarithm prior to statistical analysis (Stepwise Multiple Regression). Several statistical comparisons were made to quantify model verification status and evaluate its validity (Thomann, 1982). Correlation coefficients, the best fit linear regression between measured and calculated phosphorus concentrations and the 95% confident limits were calculated. The standard error of the estimation (root mean square error) was used as a measure of the error between the model and the observed data. In addition, the Mann–Whitney non-parametric test (Siegel and Tukey modification for variance) (Conover, 1980) was used to examine if the difference in dispersion of the standard error between models was significant.

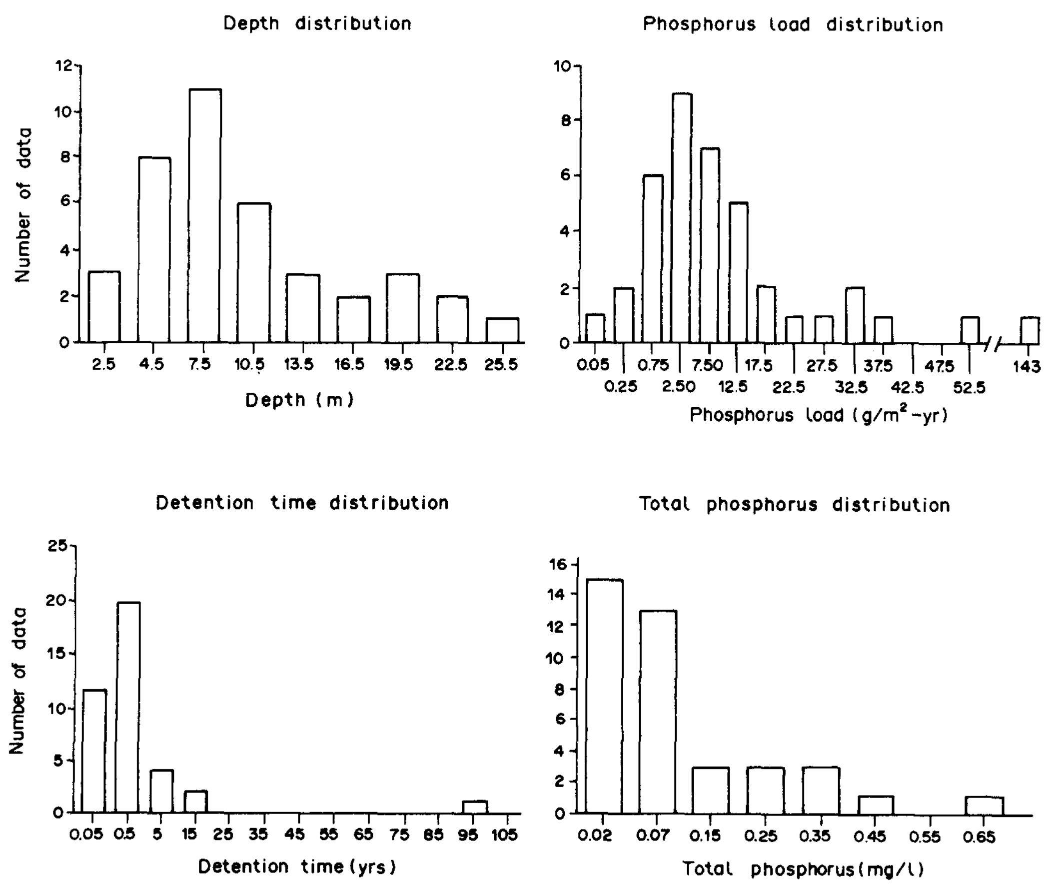

Fig. 1. Histograms of observed data.

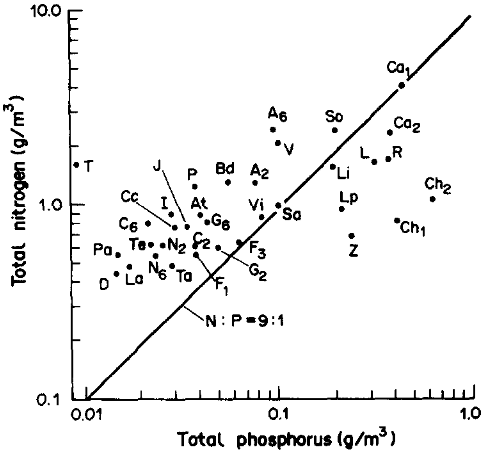

Fig. 2. Nutrient limitation.

LIMITING NUTRIENT CONCEPT

The limiting nutrient concept is based on the premise that, under a given cell stoichiometry of aquatic plants, the nutrient that will control the maximum amount of plant biomass is the nutrient that is exhausted first, or reaches a minimum before other nutrients relative to that stoichiometry. The total nitrogen (N_λ) to total phosphorus (P_λ) ratio proposed by Vollenweider (1983) of 9:1 for phytoplankton was utilized. Consequently, lakes with N_λ to P_λ ratios greater than 9 were considered to be potentially phosphorus limited whereas those with ratios less than 9 were nitrogen limited. As can be seen in Fig. 2 the majority of the lakes/reservoirs of the regional program were limited by phosphorus although in some cases other factors, such as light, could be limiting since the observed levels of both nitrogen and phosphorus were relatively high surpassing levels traditionally considered to be limiting.

TROPHIC STATE CLASSIFICATION SYSTEM

For the trophic state classification of lakes, the strategy utilized by the Organization for Economic Cooperation and Development (OECD) was applied, which provides a probabilistic quantitative framework for the different trophic state categories. Measured data of key parameters, such as phosphorus, nitrogen and chlorophyll *a*, are related to the trophic state assigned according to qualitative perceptions, as reported by Vollenweider and Kerekes (1981) for temperate lakes.

In the CEPIS Regional Program, the tentative trophic state classification supplied by the different country investigators following similar criteria (aesthetic perceptions, algal blooms, presence of macrophytes, etc.), as proposed by Vollenweider (1983) (see Table 4), and the observed total phosphorus data for each trophic state category (hypereutrophic, eutrophic, mesotrophic, oligotrophic and ultra-oligotrophic) as shown in Table 1, were used.

A normal distribution of log-transformed total phosphorus data was assumed and the following equation applied:

$$Y = \frac{1}{\sigma_s\sqrt{2\pi}}\, e^{-1/2(X-\mu)^2/\sigma_s^2} \qquad (1)$$

where

Y = log-normal probability distribution
σ_s = standard deviation (SD)
μ = average
X = parameter (i.e. log-total phosphorus, P_λ).

The quantitative results are presented in Table 5.

The Kolmogorov–Smirnov test (Blank, 1980) verified the log-normal distribution of the total phosphorus data for each of the above trophic state categories.

According to these results, the geometric means of each of the trophic categories were a multiple of approx. 2.6 for warm-water tropical lakes. Vollenweider and Kerekes (1981) observed that for temperate lakes the P_λ geometric means for the trophic state categories were multiples of about 3 and that the standard deviations were very similar for a much larger data base.

Table 4. Trophic characterization of lakes and reservoirs

Limnological characterization Categories	Ultra-oligotrophic	Oligotrophic	Mesotrophic	Eutrophic	Hypertrophic
Biomass	Very low	Low	Medium	High	Very high
Green and/or blue–green algae fraction	Low	Low	Variable	High	Very high
Macrophytes	Low or absent	Low	Variable	High or low	Low
Production dynamics	Very low	Low	Medium	High	High, unstable
Oxygen dynamics					
Epilimnic	Normally saturated	Normally saturated	Variably over-saturated	Often over-saturated	Very unstable varying from high over-saturation to complete lack
Hypolimnic	Normally saturated	Normally saturated	Variably under-saturated	Under-saturated to complete depletion	
Impairment of multi-purpose uses	Low	Low	Variable	High	Very high

Taken from Vollenweider (1983).

Table 5. Trophic state classification system quantitative results

Classification	Geometric mean, P ($mg\,m^{-3}$)	Log mean ± SD	Number of lakes	Lake identification
Eutrophic	118.7	2.074 ± 0.316	16	A_2, A_6, B_b, B_c, F_2, F_3, G_2, L, L_g, L_i, L_p, P, R, S_2, V_i, Z
Mesotrophic	39.6	1.598 ± 0.137	9	B_a, B_d, C_2, F_1, G_6, I, S_{13}, T_a, T_c
Oligotrophic	21.3	1.328 ± 0.165	10	A_t, C_c, C_6, D, J, L_a, N_2, N_6, P_a, T

Using the eutrophic classification category as a base and applying a multiple of 2.6, the average values for the other trophic states were calculated inclusive of the hypereutrophic and ultraoligotrophic classifications, since the data for these latter two were insufficient for direct calculation. Also, an average value of 0.206 for the standard deviation was applied to all distributions (σ_s not significantly different among categories, $P < 0.05$).

The conditional probability distribution for the trophic state categories, as shown in Fig. 3, results from the application of the Bayes' formula (Blank, 1980) to each of these log-normal distributed categories in the following manner:

$$P(\mathrm{CT}_i/X) = \frac{P(\mathrm{CT}_i)\cdot P(X/\mathrm{CT}_i)}{\Sigma P\,(\mathrm{CT}_i)\cdot P(X/\mathrm{CT}_i)} \tag{2}$$

where

$$\frac{P(X/\mathrm{CT}_i)}{\Sigma P(X/\mathrm{CT}_i)} = \frac{Y(\mathrm{CT}_i)}{\Sigma Y(\mathrm{CT}_i)}$$

CT_i = trophic state categories
X = logarithm of P_λ
$P(X/\mathrm{CT}_i) = Y(\mathrm{CT}_i)$ = normal distribution of probability [Y(HE) (hypereutrophic), Y(E) (eutrophic), Y(M) (mesotrophic), Y(O) (oligotrophic) and Y(UO) (ultraoligotrophic)]
$\Sigma Y(\mathrm{CT}_i)$ = sum of all the distributions.

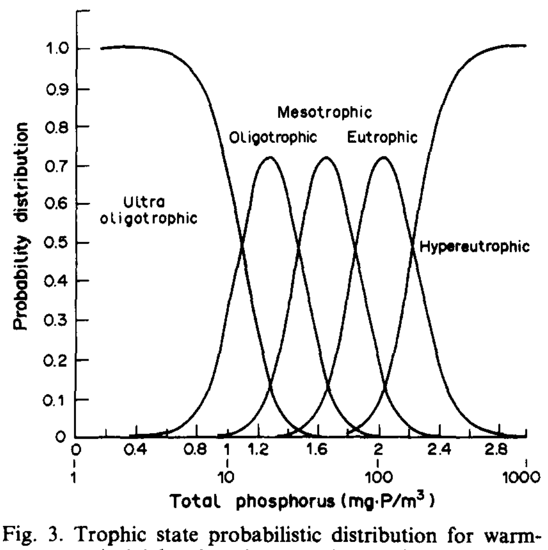

Fig. 3. Trophic state probabilistic distribution for warm-water tropical lakes based on total phosphorus [adapted from Vollenweider and Kerekes (1981) with warm-water tropical lake data].

The use of a trophic state classification system for lakes/reservoirs not only has a scientific interest, but also a managerial application, since the desired water uses are closely dependent of the trophic state and consequent water quality. In this manner, given a total phosphorus concentration and using Fig. 3, the percent probability of the trophic state of a warm-water tropical lake/reservoir can be determined.

MODEL DEVELOPMENT

The simplified models tried in the CEPIS Regional Program to predict the trophic response of lakes/reservoirs to changes in their total phosphorus concentrations were based on two kinds of relationships:

(a) The mass balance equation:

$$\mathrm{P}_\lambda = \frac{L(\mathrm{P})}{\bar{Z}(1/T_\omega + K_s)} \tag{3}$$

where

P_λ = total phosphorus ($mg\,l^{-1}$)
$L(\mathrm{P})$ = areal total phosphorus loading rate ($g\,m^{-2}\,yr^{-1}$)
$\bar{Z}$ = average depth of lake (m)
T_ω = detention time (yr)
K_s = overall loss rate of total phosphorus (yr^{-1}).

The difficulty of using equation (3) is that the overall loss rate, K_s, is not readily known nor can it be measured in a direct experimental way. Normally, the K_s term is calculated by applying equation (3) knowing the other parameters. Subsequently, correlations are estimated with more readily obtainable parameters which upon substitution permits the use of equation (3) as a predictive tool.

(b) Totally empirical equations applying stepwise multiple linear regression analyses:

$$P_\lambda = f\,[L(\mathrm{P}),\, T_\omega,\, \bar{Z}] \tag{4}$$

$$\mathrm{P}_\lambda = \mathrm{CONS}\cdot L(\mathrm{P})^{\mathrm{A}}\cdot T_\omega^{\mathrm{B}}\cdot \bar{Z}^{\mathrm{C}} \tag{5}$$

where CONS, A, B and C are constants.

With the OECD temperate lake data, Vollenweider (1976) reported significant correlations with the following relationships:

$$K_s = f(\bar{Z}) \tag{6}$$

$$K_s = f(T_\omega). \tag{7}$$

Kenney (1982) questioned the validity of equation (6) because of its inherent spurious self-correlation. In the CEPIS Regional Program, no significant correlations were achieved with equation (6) and it was abandoned (Salas and Limón, 1985).

Another form of expressing the mass balance equation (3) is as follows:

$$P_\lambda = \frac{P_{\lambda\text{in}}}{T_\omega(1/T_\omega + K_s)} \tag{8}$$

where

$P_{\lambda\text{in}}$ = the average influent total phosphorus (mg l^{-1})

Solving for K_s results in the following:

$$K_s = \frac{1}{T_\omega}\left(\frac{P_{\lambda\text{in}}}{P_\lambda} - 1\right). \tag{9}$$

As such, a correlation of K_s with T_ω [see equation (7)] could be of a spurious nature. To avoid this, the correlation analysis was applied as follows:

$$K_s = f(\text{TEMP}) \tag{10}$$

where

$$\text{TEMP} = \left(\frac{P_{\lambda\text{in}}}{P_\lambda} - 1\right).$$

The following relationship was obtained:

$$\text{TEMP} = 1.85\, T_\omega^{0.420} \tag{11}$$

with

r^2 = coefficient of determination = 0.58

SE = standard error of the estimate = 1.34

N = number of data sets = 39 (27 water bodies).

Therefore:

$$K_s = \frac{1}{T_\omega}(\text{TEMP}) = 1.85/T_\omega^{0.580} \tag{12}$$

which could be substituted ($P < 0.05$) by:

$$K_s = 2/\sqrt{T_\omega}. \tag{13}$$

This relationship is double the magnitude of that developed for the OECD temperate lakes, $K_s = 1/\sqrt{T_\omega}$ (Vollenweider, 1976) such that the calculated K_s for a warm-water tropical lake would be essentially double that for a temperature lake with an identical detention time. This result approximates that calculated theoretically by Castagnino (1982) as a function of higher temperatures and phytoplankton growth rates expected in warm-water tropical lakes. However, it should also be noted that most of the CEPIS program's waterbodies are reservoirs, and therefore a higher sedimentation rate could be expected.

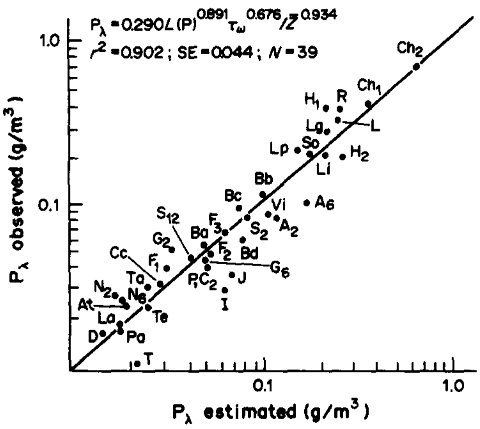

Fig. 4. Comparison of observed and calculated total phosphorus.

Substitution of equation (13) into the mass balance P_λ budget model [equation (3)] results in:

$$P_\lambda = \frac{L(P)}{\bar{Z}/T_\omega(1 + 2\sqrt{T_\omega})}. \tag{14}$$

Kenney (1988) has shown that steady state is only achieved after a time equal to $5T_\omega$ and, therefore, the steady-state assumption should be cautiously reviewed for lakes with long detention times, especially in the calculation of the K_s parameter. As can be seen in Fig. 1, 32 of the 39 data sets used in the CEPIS Regional Program had detention times of less than 1 year. Also, exclusion of the extreme data set of Lake Tequesquitengo with a $T_\omega = 98.5$ years did not significantly change ($P < 0.05$) [equation (12)]. Nevertheless, the authors recognize the validity of the concerns of Kenney (1988) for lakes with long detention times.

The best fit multiple regression equation for total phosphorus (totally empirical approach) after testing all the available independent variables for significant correlation is (see Fig. 4):

$$P_\lambda = 0.290\, L(P)^{0.891}\, T_\omega^{0.676}/\bar{Z}^{0.934} \tag{15}$$

with

$$r^2 = 0.902; \quad \text{SE} = 0.044; \quad N = 39.$$

The intercorrelation among the independent variables is not significant as demonstrated by the correlation matrix in Table 6.

With the aim to simplify and facilitate its graphical presentation, equation (15) was reorganized. The coefficient values obtained were modified within the

Table 6. Correlation matrix

	Correlation coefficients		
Variables	L(P)	T_ω	$\bar{Z}$
L(P)	1.000	0.673	0.144
T_ω		1.000	0.248
$\bar{Z}$			1.000

Table 7. Statistical comparison of total phosphorus equations with observed data sets

Model equations	N	Correlation coefficient	Error estimates S	Error estimates SE
Latin American data sets				
$P_\lambda = 0.290\, L(P)^{0.891}\, T_\omega^{0.676}/\bar{Z}^{0.934}$	39	0.914	0.983	0.0484
$P_\lambda = \frac{L(P)}{\bar{Z}}\cdot\frac{T_\omega^{3/4}}{3}$	39	0.915	0.882	0.0477
$P_\lambda = \frac{L(P)}{\bar{Z}/T_\omega(1+2\sqrt{T_\omega})}$	39	0.909	0.896	0.0569
African data sets				
$P_\lambda = 0.290\, L(P)^{0.891}\, T_\omega^{0.676}/\bar{Z}^{0.934}$	15	0.885	0.877	0.0848

Correlation between calculated and measured total phosphorus concentrations for the different model equations. Error estimates include the slope (S) of the regression line and standard error or root mean square error (SE).

limits of their confidence intervals (Student's t-test, $P < 0.05$) to obtain the following relation:

$$P_\lambda = \frac{L(P)}{\bar{Z}}\cdot\frac{T_\omega^{3/4}}{3}. \qquad (16)$$

To test the ability of the equations to accurately predict phosphorus levels, correlations between calculated and measured total phosphorus concentrations were compared showing high coefficients in all cases (see Table 7). The differences between the three standard errors are not significant (Mann–Whitney test, $P < 0.05$). Phosphorus concentrations estimated by the three equations are unbiased over the whole range of measured values [slope coefficients (S) are not significantly different from 1].

Figure 5 presents the relationship of observed phosphorus areal loading [L(P), g m^{-2} yr^{-1}], average depth ($\bar{Z}$, m) and detention time (T_ω, yr) for the data sets used to develop the total phosphorus (P_λ, g m^{-3}) models. Superimposed on this figure is the 3/4 law [equation (16)] for the fixed limits of 0.030 and 0.070 mg l^{-1} which approximately separate and oligotrophic/mesotrophic and mesotrophic/eutrophic classifications, respectively, as taken from the intercept of their respective curves in Fig. 3. Applying the OECD loading concept, one may use this graph to rapidly estimate the trophic state of a given warm-water tropical lake/reservoir using the above three independent parameters.

It is noted that both the classification numerical boundaries and mathematical models developed for the warm-water tropical lakes differ significantly from those developed for temperate lakes.

MODEL VERIFICATION

The ability of the CEPIS empirical model to predict total phosphorus concentrations was tested against data from other warm-water tropical water bodies of Africa (Water Research Commission and National Institute for Water Resources, 1980). Figure 6 presents a comparison of the observed and calculated total phosphorus concentrations resulting from the application of equation (15) to the 15 data sets from the African continent. As can be seen in this figure, the performance of the CEPIS model is deemed quite good which is further confirmed by the high correlation achieved between the measured and calculated phosphorus values, whose slope is not significantly different from the CEPIS models, as presented in Table 7.

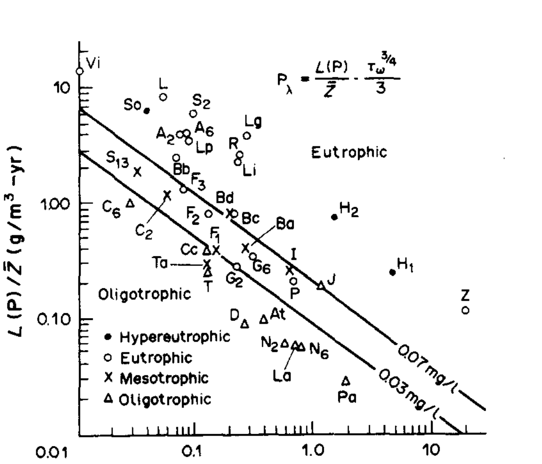

Fig. 5. $L(P)/\bar{Z}$ vs T_ω of observed data from Latin America and trophic categories qualitatively defined.

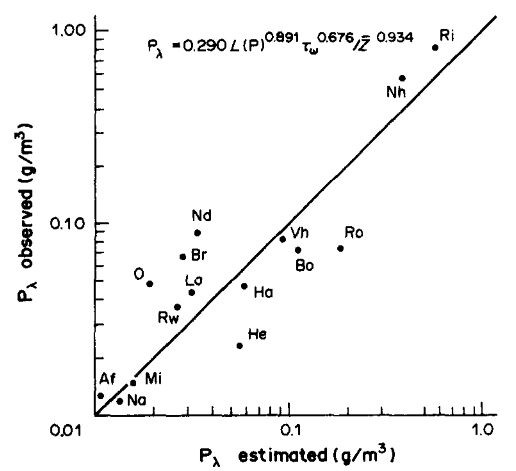

Fig. 6. Comparison of calculated [equation (15)] and observed (water bodies of Africa) total phosphorus concentrations.

Table 8. Comparison of trophic state classifications of African warm-water reservoirs (taken from Water Research Commission and National Institute for Water Research, 1980)

Lake–Dam	Symbol	Classification reported	CEPIS classification (% probability)				
			UO	O	M	E	HE
Bospoort	Bo	E	0	2	44	51	3
Bronkhorstpruit	Br	M	0	3	53	42	2
Loskop	Lo	E*	0	14	73	13	0
Olifantsnek	O	O–M	0	11	73	16	0
Rust der Winter	Rw	M	1	32	61	6	0
New Doringpoort	Nd	O–M	0	1	31	64	4
Rietvlei	Ri	HE	0	1	36	60	3
Roodeplaat	Ro	E	0	1	40	57	2
Albert Falls	Af	O	43	55	2	0	0
Hazelmere	Ha	—	0	13	72	15	0
Henley	He	O–M	6	58	36	0	0
Midmar	Mi	O	37	57	6	0	0
Nagle	Na	O	52	47	1	0	0
Vernon Hoper	Vh	E	0	1	38	59	3
Nahoon	Nh	E	0	0	0	2	98

HE, Hypereutrophic; E, eutrophic; M, mesotrophic; O–M, oligo-mesotrophic; O, oligotrophic; UO, ultraoligotrophic; —, no classification reported. *Based on phytoplankton content (nutrient content was lower).

The CEPIS probabilistic trophic state classification system based on total phosphorus was then applied to the African reservoirs and compared to the trophic state classifications reported by the African investigators (Water Research Commission and National Institute for Water Research, 1980) as shown in Table 8. In Fig. 7, the reported trophic state classifications were also superimposed on Fig. 5 and, as such, compared to the application of the 3/4 law [equation (16)] with fixed numerical boundaries for trophic state classifications. The general concurrence of the reported trophic state categories with those estimated by the CEPIS trophic state classification system prove both its universality and usefulness as a predictive tool to determine the trophic state of any given warm-water tropical lake within the range of values of the data set used for the model development (Table 3).

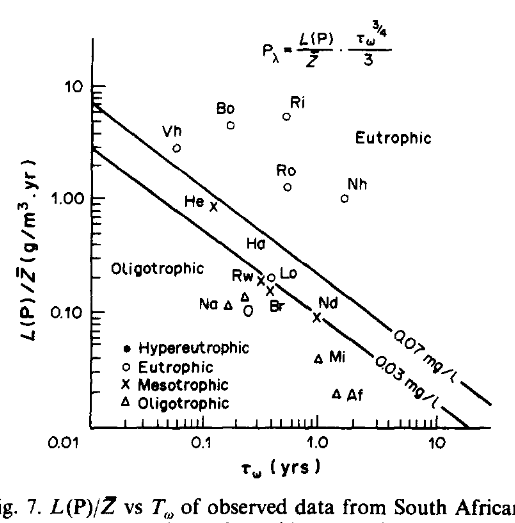

Fig. 7. $L(P)/\bar{Z}$ vs T_ω of observed data from South African warm-water reservoirs and trophic categories qualitatively defined.

CONCLUSIONS AND RECOMMENDATIONS

Using the nitrogen to phosphorus ratio concept, the majority of the warm-water tropical lakes and reservoirs of the CEPIS Regional Program, were determined to be phosphorus limiting, although in some cases other factors, such as light, could be limiting in view of the high observed levels of both nitrogen and phosphorus.

A conditional probabilistic distribution trophic classification system has been developed for the warm-water tropical lakes of the region based on total phosphorus, whose numerical boundaries differ significantly from those applied for temperate lakes. This system has been verified with warm-water tropical lakes of the African continent.

A simplified total phosphorus model has been developed which has been successfully compared to data from other warm-water tropical lakes of the African continent. As such, this model in its present form, is considered to be a useful tool in the management and planning of water resources.

Acknowledgements—The direction and consultation provided to the program by Dr Richard A. Vollenweider is appreciated as is the advice given by Beatríz Castañeda on statistical matters. We also thank all the countries that sent their data, without which the CEPIS Regional Program could not have been successfully executed.

REFERENCES

American Public Health Association (APHA), American Water Works Association (AWWA) and Water Pollution Control Federation (WPCF) (1971, 1976, 1981 and 1985) *Standard Methods for the Examination of Water and Waste Water*, 13th, 14th, 15th and 16th editions. Washington, D.C.

Blank L. (1980) *Statistical Procedures for Engineering, Management and Science.* McGraw-Hill, New York.

Castagnino W. A. (1982) Investigación de modelos simplificados de eutroficación en lagos tropicales. Pan American Health Organization (PAHO), Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS). Revised version.

Centro de Estudios Limnológicos (CEL) (1985) Protocol of Lago Chapala and Laguna de Cajititlán related to the CEPIS Regional Project on Eutrophication of Warm-Water Tropical Lakes.

Centro de Estudios Limnológicos (CEL) (1987) *IV Meeting of the CEPIS Regional Project on Eutrophication of Warm-Water Tropical Lakes*; Report.

Centro de Estudios Limnológicos (CEL) (1988) Data sent to CEPIS with the communication of 26 September 1988.

Companhia de Agua e Esgotos de Brasília (CAESB) (1982) Investigaçao de modelos simplificados de eutrofizaçao em lagos tropicais. Relatório 1 (Vols 1 and 2). Dados do Lago Paranoá, Anexo A, Balance Hídrico do Lago Paranoá, Projeto de Cooperaçao Técnica CAESB/CEPIS.

Companhia de Agua e Esgotos de Brasília (CAESB) (1983) Información sobre el Lago Descoberto entregada en el Segundo Encuentro del Proyecto Regional de Eutroficación en Lagos Tropicales.

Companhia de Tecnologia de Saneamento Ambiental (CETESB) (1985) Tercer Encuentro Regional de Eutrofizaçao de Lagos Tropicais; Relatório Técnico, São Paulo.

Companhia de Tecnologia de Saneamento Ambiental (CETESB) (1987) *IV Meeting of the CEPIS Regional Project on Eutrophication of Warm-Water Tropical Lakes*; Relatório Técnico, São Paulo.

Conover W. J. (1980) *Practical Non-Parametric Statistics*, 2nd edition. Wiley, New York.

Cordeiro Netto O. and Dutra Filho P. (1981) O aporte de fósforo ao Lago Paranoá. Companhia de Agua e Esgotos de Brasília, XI Congresso Brasileiro de Engenharia Sanitária e Ambiental.

Corporación Autónoma Regional del Cauca (CVC) (1988) Letter SRN-DAG-J-306-88 sent with data to CEPIS.

Dirección de Investigación de Ambiente (DIA) (1979) Estudio integral sobre la contaminación del Lago de Valencia, Informe No. 1, estudio limnológico, primera etapa. Caracas, Ministerio del Ambiente y de los Recursos Naturales Renovables.

Environmental Quality Board of Puerto Rico (EQB) and Center for Energy and Environmental Research (1984) An environmental evaluation of La Plata Lake, Toa Alta, Puerto Rico. U.S. Environmental Protection Agency.

Fundaçao Estadual de Engenharia do Meio Ambiente (FEEMA) (1982) Caracterizaçao da área de estudo—Reservatório de Funil. Letter 07/82 sent to CEPIS.

Fundaçao Estadual de Engenharia do Meio Ambiente (FEEMA) (1987) *IV Meeting of the CEPIS Regional Project on Eutrophication in Warm-Water Tropical Lakes*; Relatório Técnico, Río de Janeiro.

Fundaçao Estadual de Engenharia do Meio Ambiente (FEEMA) (1989) Letter sent with data to CEPIS.

Hydroscience Inc. (1976) Eutrophication analysis of the Lake Livingston Reservoir. Texas Water Quality Board, Austin, Tex.

Instituto Mexicano de Tecnología del Agua/Secretaría de Agricultura y Recursos Hidráulicos (IMTA/SARH) (1986) *IV Meeting of the CEPIS Regional Project on Eutrophication in Warm-Water Tropical Lakes*; Report.

Instituto Mexicano de Tecnología de Agua/Secretaría de Agricultura y Recursos Hidráulicos (IMTA/SARH) (1988) Letter sent with data to CEPIS.

Kenney B. C. (1982) Beware of spurious self-correlations. *Wat. Resour. Res.* **18,** 1041–1048.

Kenney B. C. (1988) On the dynamics of phosphorus in lake systems. National Hydrology Research Institute, Sakatchewan (Draft).

Lin C. J. (1981) Integrated investigation of limnology and pollution in Lake Valencia, Part II. Nutrients, pollutant inputs and dispersion. Caracas, Ministerio del Ambiente y de los Recursos Naturales Renovables.

OECD (1982) Eutrophication: monitoring assessment and control. Organization for Economic Cooperation and Development, Paris.

Quiñones Marques F. (1980) Limnology of Lago Loiza, Puerto Rico. U.S. Geological Survey and Commonwealth of Puerto Rico, Water Resources Investigation, pp. 79–97.

Rast W. and Lee G. F. (1978) Evaluation of nutrient load estimates for lakes; summary analysis of the North American U.S., portion of O.E.D.C. Eutrophication Project. U.S. EPA Report, EPA/3-78-008.

Salas H. J. (1983) *Proceedings of the Second Meeting of the Regional Project on Eutrophication of Warm-Water Tropical Lakes.* Pan American Health Organization (PAHO), Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS).

Salas H. J. and Limón G. (1986) *Proceedings of the Third Meeting of the Regional Project on Eutrophication of Warm-Water Tropical Lakes.* Pan American Health Organization (PAHO), Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS).

Salas H. J. and Martino P. (1988) *Proceedings of the Fourth Meeting of the Regional Project on Eutrophication of Warm-Water Tropical Lakes.* Pan American Health Organization (PAHO), Pan American Center for Sanitary Engineering and Environmental Sciences (CEPIS).

Sonzogni W. C., Jonteith T. J., Gock W. N. and Hugues V. G. (1978) United States Great Lakes tributary loadings. U.S. EPA and U.S. Task D. Committee.

Thomann R. V. (1982) Verification of water quality models. *J. envir. Engng Div., Proc. Am. Soc. civ. Engrs* **108,** 923–940.

U.S. Geological Survey (1978) Limnology of Laguna Tortuguero, Puerto Rico. WRI 77–122.

Vásconez G. J. E. (1983) Poza Honda. Segundo Encuentro de Eutroficación en Lagos Cálidos. Instituto Ecuatoriano de Obras Sanitarias/Centro de Rehabilitación de Manabí.

Vollenweider R. A. (1968) Scientific fundamentals of the eutrophication of lakes and flowing waters, with particular reference to nitrogen and phosphorus as factors in eutrophication. OECD, Paris. Tech. Report No. DAS/CSI/68.27.

Vollenweider R. A. (1976) Advances in defining critical loading levels for phosphorus in lake eutrophication. *Mem. Inst. Ital. Idrobiol. Bott.* Marco de Marchi, **33,** 53–83.

Vollenweider R. A. (1983) Eutrophication. Notes distributed during the *II Meeting of the Regional Project on the Eutrophication of Tropical Lakes.*

Vollenweider R. A. and Kerekes J. J. (1981) Background and summary results of the OECD cooperative program on eutrophication. In *Restoration of Lakes and Inland Waters.* EPA/440/5-81-010: 25–36.

Water Research Commission and National Institute for Water Research (1980) The limnology of some selected South African impoundments, Pretoria.